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# PROBLEM OF REDUCING HYDRODYNAMIC DRAG

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16. Abstract

A mathematical analysis of the relationship of hydrodynamic drag to planar and turbulent flow in a viscous fluid is presented. Based upon observations of the movements of dolphin and tuna, the Grey and d'Alembert-Stokes paradoxes are considered in a new set of vector equations. The effects of limiting conditions and boundary-layer behavior are reviewed relative to streaming waves. Periodicity of the boundary layer is related to periodic boundary conditions. The relationship of streaming waves on elastic properties is also discussed.

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## PROBLEM OF HYDRODYNAMIC DRAG

V. I. Merkulov

### The Grey and d'Alembert-Stokes Paradoxes /52*

Thirty years ago the English physiologist Grey, observing that dolphins swim rapidly, and in investigating their muscle-capacity, reached a paradoxical conclusion: that as they move, the animals need to develop an energy seven times greater than is necessary to move their muscles. All this contradicts previously known facts.

Experiments to clarify this paradox of Grey's force one to make the assumption that, in swimming, the live dolphin experiences far less drag than a solid model of it, upon which basis the English physiologist came to his conclusion.

Some investigator has observed that Grey's calculations were exaggerated: the dolphin experiences a drag four times less than the solid model. Therefore the physiologist erred in evaluating the energy of the moving muscles. Perhaps there was no paradox at all.

Instead of looking for answers to these questions, we made simple calculations of the energy necessary for tuna, which move at a velocity of 90 km/hr. We intentionally chose this animal and not, say, the swordfish, which can swim 130 km/hr, because the velocity the tuna attains is instrumentally recordable.

* Numbers in the margin indicate pagination in the foreign text.

Let us suppose that the tuna experiences the same drag as the solid model. Then for a 100% coefficient of effective motor activity, an energy

$$N = c\,\frac{\rho v^3 d^2 \pi}{8}.$$

is necessary.

With a cross-sectional diameter of d = 0.5 m and a movement rate of v = 25 m/sec, N = 100 kcal.

For this estimate, we took as the experimental value of the dimensionless coefficient c for bodies 2.5 m long the Reynolds number $6 \cdot 10^7$.

In order to develop so great an energy, the tuna must take in as much oxygen as a hundred horses. Even if one is not a physiologist, it is not hard to understand that for a fish which breathes oxygen, which is soluble in water in very small concentrations, this is absolutely impossible.

Therefore the evidence in nature of an effective mechanism of decreasing hydrodynamic drag has not caused any doubt. Engineers have always thought about decreasing the hydrodynamic drag of bodies which move underwater. Studying the fast-moving inhabitants of rivers and oceans yields a basis for supposing that in this way there is even an inexhaustible source of energy.

The paradox of the physiologist Grey is answered with the theorem formulated by the mathematicians d'Alembert and Stokes, which in due time, came to be known as the equation of motion of a viscous fluid.

These investigators noted this fact: when drag in a /53
fluid devoid of viscosity equals zero, in low-viscosity fluids it is quite large.

Noting that the viscosity is very low, the conclusion may be drawn that the removal of the d'Alembert-Stokes paradox simultaneously raises the conclusion of Grey.

It is just for this reason that we shall propose a new set-up for the problem of the dynamics of a viscous fluid.

## The New Problem Set-Up

Motion of a viscous incompressible fluid may be described by equations which we shall write in the dimensionless vector form:

$$(\bar{\omega}\nabla)\bar{\omega} = -\frac{1}{\rho}\nabla p + \frac{1}{R}\Delta\bar{\omega}; \quad \operatorname{div}\bar{\omega} = 0. \tag{1}$$

Here $\bar{w}(u,v,w)$ is the velocity vector, p is pressure, $\rho$ = a constant, the density of the fluid, and $R = \frac{vl}{\nu}$ is the Reynolds number.

If, in a fluid immovable at infinity, a body with a surface S moves with a uniform velocity in the direction of the x axis, the equation becomes, under the following limiting conditions:

$$v = w = 0, \quad u = 1 \text{ на } S, \tag{2}$$

$$\lim u = \lim v = \lim w = 0 \tag{3}$$

Having taken R = ∞, we obtain the degenerate equations:

$$(\bar{\omega}_0\nabla)\bar{\omega}^0 = -\frac{1}{\rho}\nabla p^0, \qquad \operatorname{div}\bar{\omega}^0 = 0, \tag{4}$$

which describes the flow of a viscous fluid. They have a low order and, instead of three limiting conditions (2) at the boundary s, only one is allowed for the normal component of the velocity

$$\omega_n = f(s) \text{ на } S. \tag{5}$$

The d'Alembert-Stokes paradox consists of the coefficient of drag c(R), calculated from the solution to problems (1)-(3), with the increase in R not passing through the zero value of this coefficient, which follows from the solution of equations (4) and (5).

In such a way, the physical problem of reducing hydrodynamic drag leads to this mathematical problem.

It is necessary to point out all the possible boundary values of the vector w at the boundary S, for which the solution to equations (1)-(3) proceeds by means of the coefficient of drag to the solution of problems (4) and (5).

With this aim, it is limited to a consideration of planar flow which can be described with the aid of the flow function by the following system of equations:

$$\frac{D(\Delta\psi, \psi)}{D(x, y)} = \frac{1}{R}\Delta^2\psi. \qquad (6)$$

For equation (6), it is necessary to give the following limiting conditions:

$$\frac{\partial\psi}{\partial s} = v_n(S) \text{ на } S, \qquad (7)$$

$$\frac{\partial\psi}{\partial n} = v_s(S), \qquad (8)$$

$$\lim \frac{\partial\psi}{\partial y} = 1; \qquad (9)$$ /54

$$\lim \frac{\partial\psi}{\partial x} = 0 \text{ при } x^2 + y^2 \to \infty. \qquad (10)$$

Here $v_n$ and $v_s$ are the normal and tangential velocity of the boundary flows.

## Turbulent Flow of an Ideal Fluid Approximating Flow of a Viscous Fluid

It always seems to be observed that the boundaries cannot be deformed, and then $v_n = v_s = 0$. We observe them to be not equal to zero, and we use them such that the solution of the boundary problems (6)-(10) are not subject to the d Alembert-Stokes paradox. Then for the coefficient of drag having continuously developed at the solution of the following problem for an ideal fluid:

$$\frac{D(\psi, c\psi)}{D(x, y)} = 0, \tag{11}$$

$$\frac{\partial\psi}{\partial s} = U_n(S) \text{ на } S, \tag{12}$$

$$\lim \frac{\partial\psi}{\partial y} = 1 \text{ при } x^2 + y^2 \to \infty. \tag{13}$$

It is known that equation (11) is equivalent to:

$$\Delta\psi = F(\psi) \tag{14}$$

The solution to equation (14), which we designate $\Psi_0$, satisfies, with an error on the order of $0(\frac{1}{R})$, equation (6) and the limiting conditions (7) and (8). It is not hard to prove that if the function $F(\Psi)$ is chosen such that lim $F(\Psi) = 0$ for $\to \infty$, then at the same time the condition (10) will be fulfilled.

Let us assume $U_s(S)$ to be such that there exists a corresponding function $F(\psi)$, for which the equation

$\frac{\partial \psi_F}{\partial n} = U_s(S)$ is obtained.

Then the solution to problems (11)-(13), with an error on the order of $0\frac{(1)}{R}$ can be the solution of the resulting problems (6)-(10) and our task is reduced to finding the function $F(\psi)$.

Let there be some resultant function $F_0$ as to the solution of $\psi_0$ of the equation

$\Delta\psi_0 = F_0(\psi)$,



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which satisfies the limiting conditions (7), (9), and (10), or does not satisfy the condition (8), such that

$$\frac{\partial\psi_0}{\partial n} - U_s(S) = \eta U_1(S),$$

where $\eta$ is a minor parameter.

Let us represent the function F in the form of a sum

$$F(\psi) = F_0(\psi) + \eta F_1(\psi).$$

Then the solution to the equation

$$\Delta\psi = F_0(\psi) + \eta F_1(\psi)$$

can also be represented in the form of a sum /55

$$\psi = \psi_0 + \eta\varphi_0.$$

For this variation in the function $\psi$ with an accuracy to the second order of smallness relative to $\eta$, the linear equation $\Delta\varphi - \varphi F_0'(\psi_0) = F_1(\psi_0)$.

is also satisfied.

It is possible to represent the arbitrary variation in the form of a series for some complete set of functions

$$F_1(\psi_0) = \sum_{n=1}^{N} a_n p_n(\psi_0).$$

Let us introduce into the investigation, the set of functions $\phi_n$, which are solutions to the following nonhomogeneous equations

$$\Delta\varphi_n - \Delta\varphi_n F_0'(\psi_0) = p_n(\psi_0)$$

with the homogeneous limiting conditions

$$\frac{\partial\phi_n}{\partial s} = 0 \text{ in } S; \lim \frac{\partial\phi}{\partial y} = 0 \text{ for } x^2 + y^2 \to \infty$$

Then the variation in the function $\psi$ has the form of a sum

$$\varphi_0 = \sum_{n=1}^{N} a_n\varphi_n,$$

where the coefficients $a_n$ are the same as in the function $F_1(\Psi)$

Now it is possible to calculate the normal derivative

$$\frac{\partial\varphi}{\partial n} = \sum_{n=1}^{N} a_n \frac{\partial\varphi_n}{\partial n}$$

and to set the requirement that it approximate the function $U_1(S)$, for instance, in the matrix $L_2$

$$\left\| \sum_{n=1}^{N} a_n \frac{\partial\varphi_n}{\partial n} - U_1(S) \right\| < \varepsilon.$$

The requirement that the expression mentioned attain its least value leads to a system of linear equations in the coefficients $a_n$

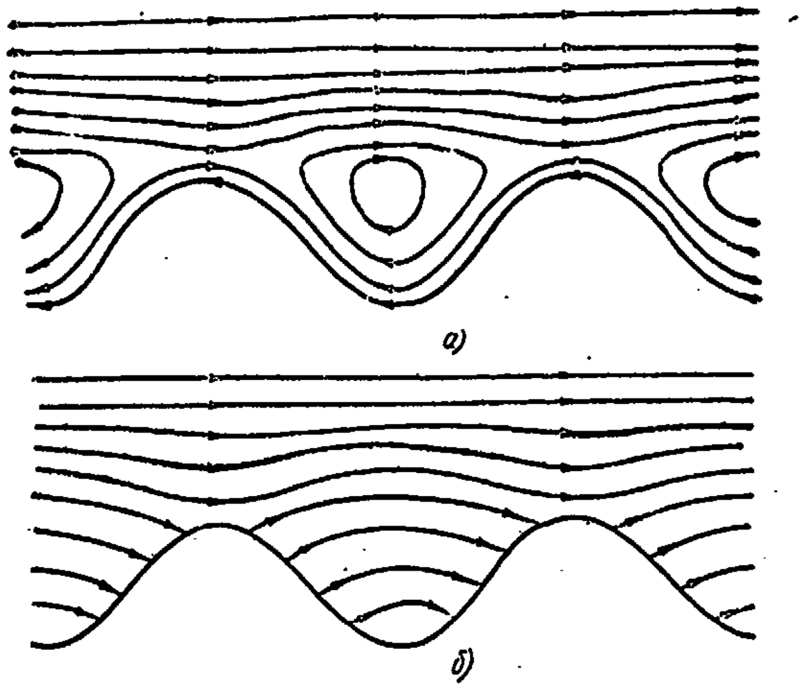
$$\sum_{n=2}^{N} a_n A_{nm} = B_m \quad (m = 2, \ldots, N),$$

where $A_{nm}$ and $B_m$ are expressed as the squares

$$A_{nm} = \int \frac{\partial\varphi_n}{\partial n} \frac{\partial\varphi_m}{\partial n} ds, \qquad B_m = \int_s \frac{\partial\varphi_m}{\partial n} U_1 ds.$$

_A priori_ investigations of the given problem in the general form are quite numerous. But the _a posteriori_ evaluations in the process of calculation yield the possibility of extimating the corresponding subsequent approximations and those which the function $U_1(S)$ can attain by the approximations selected.

It is usually more interesting if the series $\sum_{n=1}^{N} a_n \frac{\partial\varphi_n}{\partial n}$ does not approach the function $U_1(S)$, although possibly it is only slightly different. In the case given, the solution approximates the motion of a viscous fluid very well in the middle of the flow, but at the boundary zone with a width on /56
the order of $0\left(\frac{1}{\sqrt{R}}\right)$, a boundary layer occurs, owing to which




all the limiting conditions are satisfied. We shall consider this in more detail for complete periodic flow.

We studied flow which occurs during motion of a viscous fluid along a streaming wave. Insofar as it achieves, in the absolute system of coordinates, nonstationary limiting conditions, we explored this flow in a carrier system of coordinates, which moves along with the crest of the wave with a uniform velocity. In this system, a streaming wave is created on an undeformed, undulating surface.

Wave flow of a viscous fluid with periodic boundary conditions will also be periodic. Therefore it is possible to look for a single long wave at the boundaries.

The velocities of the outer stream and the boundary currents have opposite directions. Wave flow of an ideal fluid, shown in the figure, ensures the necessary velocity agreement.



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This flow, called secondary, is described by the solution to the equation

$$\Delta\Psi_0 = F_0(\Psi),$$

where

$$F_0(\psi_0) = \gamma(1 - \mathrm{th}\,\beta\psi_0).$$

is assumed.

It is not hard to understand that such a solution, being approximate, is not unique. However, it can serve as a first approximation for applying a preliminary algorithm, which yields the possibility of finding the function, the normal derivative of which best approximates the value of the tangential velocity $v_s$ of the boundary currents.

In this general case, if $v_s \neq \frac{\partial\psi_0}{\partial n}$, the solution obtained can only be used in the middle of the flow. But in a narrow zone with a width on the order of $0\left(\frac{1}{\sqrt{R}}\right)$, a boundary layer /57 is created, owing to which the necessary solution on the boundary is achieved.

We found the solution at the boundaries of a viscous boundary zone. For a system of equations, we take this limiting arc x and the normal to it y.

## The Periodic Boundary Layer

Flow in a boundary layer is described by the Prandtl equations:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{R}\frac{\partial^2 u}{\partial y^2}; \quad (15)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (16)$$

$$\frac{\partial p}{\partial y} = 0. \quad (17)$$

Here u and v are the velocity components in the curvilinear system of coordinates chosen.

The functions $v_s$ and $\frac{\partial \psi_0}{\partial n}$ will be limiting values of the velocity for the component u. It will be convenient for us to display them as a Fourier series:

$$u(x, 0) = v_s = 1 + \sum_{n=0}^{\infty} a_n \cos nx, \tag{18}$$

$$u(x, \infty) = \frac{\partial \psi^0}{\partial n} = 1 + \sum_{n=0}^{\infty} a'_n \cos nx. \tag{19}$$

Here we observe that the choice of scale for the wavelength is reduced to $2\pi$.

Owing to $\frac{\partial p}{\partial y} = 0$, the pressure p coincides with the pressure in the outer stream and satisfies the equations

$$u^0 \frac{\partial u^0}{\partial x} + v^0 \frac{\partial u^0}{\partial y} = \frac{1}{R} \frac{\partial^2 u^0}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x}.$$

We introduce a new dependent variable $\bar{u} = u - u^0$. Substituting this function in (15), we obtained the homogeneous equation

$$\bar{u}^0 \frac{\partial \bar{u}}{\partial x} + \bar{u} \frac{\partial u^0}{\partial x} + \bar{u} \frac{\partial \bar{u}}{\partial y} + v \frac{\partial u^0}{\partial y} + v \frac{\partial \bar{u}}{\partial y} = \frac{1}{R} \frac{\partial^2 \bar{u}}{\partial y^2}. \tag{20}$$

We reject values of a second order of smallness; then

$$\frac{\partial u}{\partial x} = \frac{1}{R} \frac{\partial^2 \bar{u}}{\partial y^2}. \tag{21}$$

This simple equation requires solution with the following limiting conditions:

$$u(x, 0) = \sum_{n=0}^{\infty} (a_n - a'_n) \cos nx, \tag{22}$$

$$\bar{u}(x, \infty) = 0. \tag{23}$$

The solution of equation (21) can be sought in the form of a series in which the coefficients $u_n$ and $w_n$ must satisfy the following system of equations: $\bar{u} = \sum_{n=0}^{\infty} u_n(y) \cos nx + v_n(y) \sin nx,$ /58

$$\frac{1}{R} \frac{d^2 u_n}{dy^2} + n v_n = 0; \quad \frac{1}{R} \frac{d^2 v_n}{dy^2} - n u_n = 0.$$

We multiply the second equation by i and add it to the first. For the complex unknown

$$w_n = u_n + i v_n$$

we obtain

$$\frac{d^2 w_n}{dy^2} - iRnw_n = 0. \tag{24}$$

The general solution is written thus:

$$w_n = c_n e^{-y\sqrt{\frac{Rn}{2}}}\left[\cos y\sqrt{\frac{Rn}{2}} - i\sin y\sqrt{\frac{Rn}{2}}\right] +$$
$$+ d_n e^{y\sqrt{\frac{Rn}{2}}}\left[\cos y\sqrt{\frac{Rn}{2}} + i\sin y\sqrt{\frac{Rn}{2}}\right].$$

From the limiting conditions at infinity, it follows that $d_n = 0$. Then for the coefficients $u_n$, $v_n$, we find the expression:

$$u_n = c_n e^{-y\sqrt{\frac{Rn}{2}}}\cos y\sqrt{\frac{Rn}{2}}; \quad v_n = -c_n e^{-y\sqrt{\frac{Rn}{2}}}\sin y\sqrt{\frac{Rn}{2}}.$$

Thus, the unknown function $\bar{u}$ will be given in the form of a series

$$\bar{u}(x, y) = \sum_{n=0}^{\infty} c_n e^{-y\sqrt{\frac{Rn}{2}}}\cos\left(nx - y\sqrt{\frac{Rn}{2}}\right).$$

From the limiting condition (22), we obtain:

$$c_n = a_n - a_n'$$

Returning to the previous unknown, we find

$$u(x, y) = 1 + \sum_{n=0}^{\infty}\left[a_n'\cos nx + (a_n - a_n')\, e^{-y\sqrt{\frac{Rn}{2}}}\cos\left(nx - y\sqrt{\frac{Rn}{2}}\right)\right].$$

Now it is possible to calculate the dimensionless behavior of forces of tangential friction, which is equal to the coefficient of friction squared

$$c = \frac{1}{\pi}\int_0^{\pi}\tau u\,dx = \frac{1}{\pi\sqrt{R}}\sum_{n=1}^{\infty}(a_n - a_n')\sqrt{n}\,a_m\int_0^{\pi}\cos nx\cos\left(mx + \frac{\pi}{4}\right)(dx = \tag{25}$$
$$= \frac{1}{2\sqrt{R}}\sum_{n=1}^{\infty}(a_n - a_n')\,a_n\sqrt{\frac{n}{2}}.$$

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The specifics of the boundary layer studied consist of causing a small sequence of velocities, directed to one side. This justifies taking its simplified equation. Moreover, periodic boundary conditions cause periodicity in the boundary layer.

Both these circumstances lead to an effective Reynolds flow number which was examined, appears low, and does not depend on the dimensions of the entire body.

## The Streaming Wave: The Mechanism of Reducing Hydrodynamic Drag /59

The solution which we constructed, owing to its complete periodicity, describes flow with zero drag in the usual sense. Having applied motion with an active boundary, the drag coefficient is then calculated from the force necessary to sustain flow with zero drag. This force combines dissipation, which is equal to

$$\frac{1}{R}\iint(\Delta\psi_0)^2,$$

and the energy necessary for the formation of a vortex in one hour. The first addition is of the order of $O(\frac{1}{R})$, and the second $O(\frac{\lambda}{L})$ where $\lambda$ is the wavelength and L the length of its body, which is flowing along. This same force must equal, in the optimal case, the work done over one hour by the forces of tangential friction, calculated from formula (25).

What advantages has the streaming wave compared with a smooth solid surface? They are conditioned first of all by the periodicity of the flow. While at the solid boundary, the local Reynolds number increases, avoiding the first critical point (such a flow necessarily becomes turbulent, it remains constant in periodic flow and may be chosen sufficiently small that the flow will be laminar.

The advantage of the flow which was studied is especially great in flowing bodies of small extent. Then, as in bodies with a solid boundary there are formed abrupt flows with a high drag coefficient, the active boundary preserves continuous flows with a zero drag to pressure. If one takes into consideration that the presently provided decrease of drag to pressure is reduced to the size of the relative length, i.e. to the size of the drag friction, it is possible to understand the advantage which the flow described yields.

All the cases cited above are prohibited in the solution of the equations of viscous-fluid motion. Using these in conformity with physical flow, it is possible, if the limiting conditions are taken for those physically achieved.

There remain some requirements for this.

Usually, it is possible to imagine a mechanism, observed in a model, which compels varying the boundary corresponding to the given law. However, such a construction is so complicated that it is practically prohibitive. Therefore, let us look at another procedure.

The streaming wave may be a regime of auto-oscillation on an elastic surface. Excitation and oscillation of the regime are realized either by a specific mechanism operating or by means of the principal stream.

In this way, creation of a streaming wave is sufficient to regulate elastic properties in relation to the velocity of the fluid flow.

It is entirely possible that such a mechanism of streaming-wave formation and hydrodynamic-drag reduction occurs, we say, in dolphins and tuna. Their thick layer of fat and muscle may be that elastic covering in which the wave is generated. Special muscles in the dolphin regulate skin tension and therefore its elastic properties.

Analysis of the equations of motion for a viscous fluid has shown that in this case, if a specially selected streaming wave is generated on the surface of a body, an original secondary flow occurs along it with zero drag. Here, such an effect is attained by loss of very small energies.

The results obtained may be used to explain the "secrets" of the rapid and economical swimming of aquatic animals and to create technical means for reducing hydrodynamic drag.